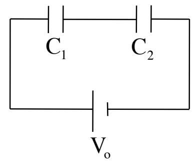
Problem 26.34



a.) In series combinations, the amount of charge on each cap is the same and will be the same as that on the equivalent capacitance, sooo . . .

$$\frac{1}{C_{equ}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

$$\Rightarrow \quad C_{equ} = \left[\frac{1}{\left(2.50\text{x}10^{-6}\text{F}\right)} + \frac{1}{\left(6.25\text{x}10^{-6}\text{F}\right)} \right]^{-1}$$

$$\Rightarrow \quad C_{equ} = 12.0\text{x}10^{-6}\text{ F}$$

b.) The energy stored in the system:

$$E = \frac{1}{2} C_{equ} V_o^{\,2}$$

$$= \frac{1}{2}\left(12.0\text{x}10^{-6}\text{ F}\right)\left(12.0\text{ V}\right)^2$$

$$= 8.64\text{x}10^{-4}\text{ J}$$

c.) In a series combination, not only do all the capacitors have the same charge on them, that charge is also the same as the charge on the equivalent capacitance. Calculated, it is:

$$Q = C_{equ} V_o$$

$$= \left(12.0\text{x}10^{-6}\text{ F}\right)\left(12.0\text{ V}\right)$$

$$= 1.44\text{x}10^{-4}\text{ C} \qquad (= Q_1 = Q_2)$$

The energy on the first cap is, therefore:

$$E = \frac{1}{2} C V_o^{\,2}$$

$$\Rightarrow \quad E_1 = \frac{1}{2} C_1 \left(\frac{Q_1}{C_1}\right)^2$$

$$= \frac{1}{2}\frac{Q_1^{\,2}}{C_1}$$

$$= \frac{1}{2}\frac{\left(1.44\text{x}10^{-4}\text{ C}\right)^2}{\left(1.80\text{x}10^{-5}\text{ F}\right)}$$

$$= 5.76\text{x}10^{-4}\text{ J}$$

And on the second cap:

$$E_2 = \frac{1}{2}\frac{Q_1^{\,2}}{C_1}$$

$$= \frac{1}{2}\frac{\left(1.44\text{x}10^{-4}\text{ C}\right)^2}{\left(3.60\text{x}10^{-5}\text{ F}\right)}$$

$$= 2.88\text{x}10^{-4}\text{ J}$$

d.) Do the individual energies sum to the total energy as calculated using the equivalent capacitance?

$$E_1 + E_2 = \left(5.76\text{x}10^{-4}\text{ J}\right) + \left(2.88\text{x}10^{-4}\text{ J}\right)$$

$$= 8.64\text{x}10^{-4}\text{ J}$$

Great jumping huzzahs, the sum matches the equivalent capacitance calculated energy!

e.) The total energy wrapped up in the equivalent cap will always equal the sum of the energies involved in the individual capacitors. That is what it *means* to be an equivalent capacitance.

f.) If the combo had been in parallel, what voltage would have been required for the system to hold that same amount of energy?

The equivalent capacitance for the parallel system would be:

$$C_{eq} = C_1 + C_2$$

$$= \left(1.80\text{x}10^{-5}\text{ F}\right) + \left(3.60\text{x}10^{-5}\text{ F}\right)$$

$$= 5.40\text{x}10^{-5}\text{ F}$$

With the energies being the same, that would mean:

$$E_{series} = E_{parallel}$$

$$\text{so} \quad \left(8.64\text{x}10^{-4}\ \text{J}\right) = \frac{1}{2}C_{eq}V_o^{\,2}$$

$$\Rightarrow \quad V_o = \sqrt{\frac{2\left(8.64\text{x}10^{-4}\ \text{J}\right)}{C_{eq}}}$$

$$= \sqrt{\frac{2\left(8.64\text{x}10^{-4}\ \text{J}\right)}{\left(5.40\text{x}10^{-5}\ \text{F}\right)}}$$

$$= \left(5.66\ \text{V}\right)$$

g.) In a parallel combination, the voltage is the same across each element. As

$$E = \frac{1}{2}CV^2$$

the large capacitance "C" will have the greater energy associated with it. (For capacitors hooked up in series, where *charge* is common and the *voltage* is related to the *inverse* of the capacitance (V=Q/C), the opposite is true.)